Statistical Properties of the Acoustic Field in Inhomogeneous Oceanic Environment

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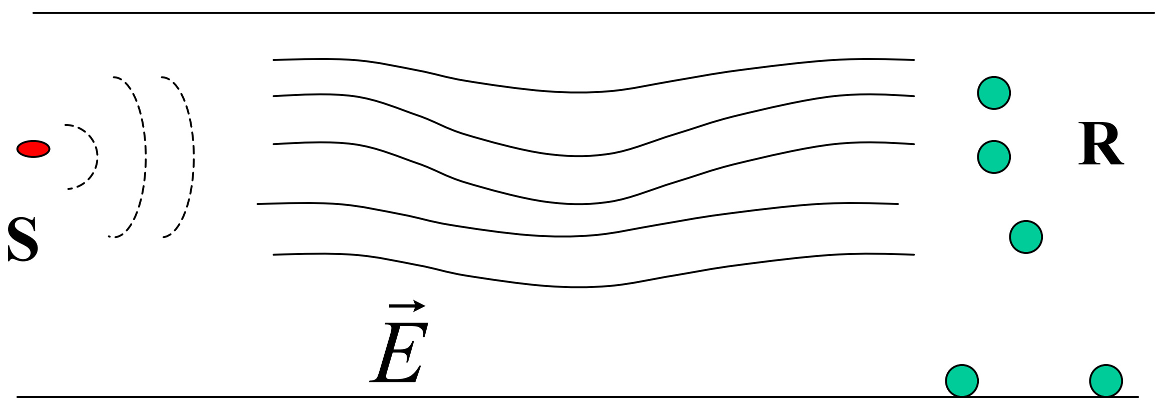
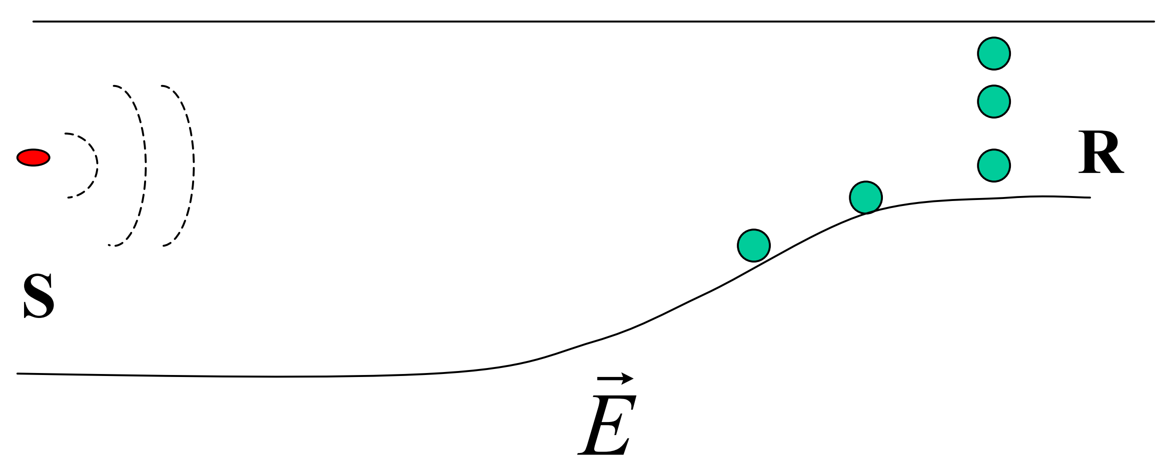
We will investigate the effects of water masses or bottom inhomogeneities

Examples:

Internal waves soliton



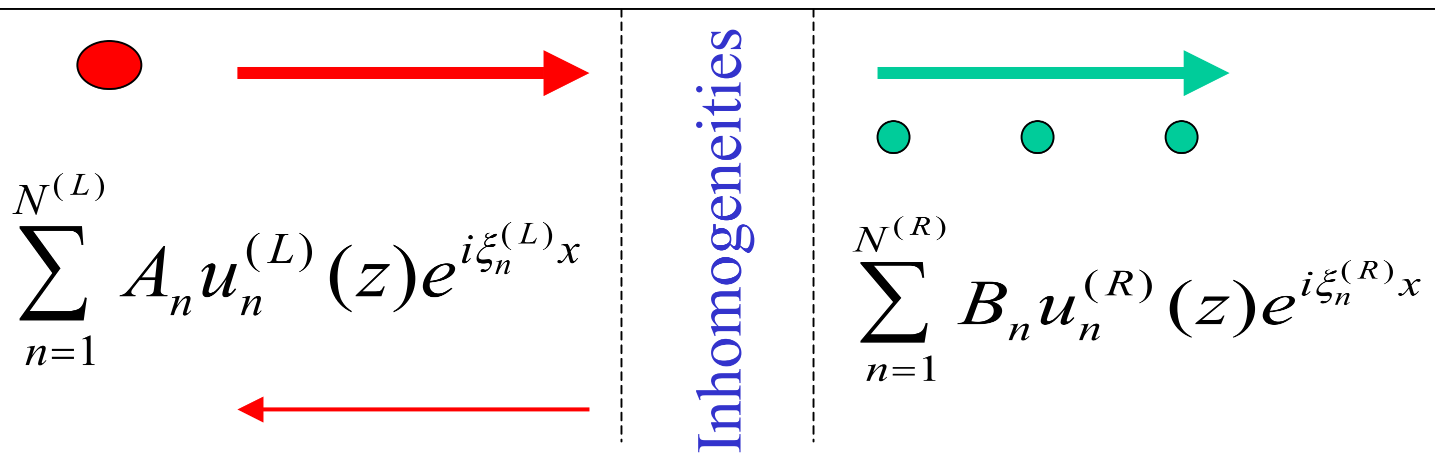
Bottom profile



Parameters known with uncertainties:

a) Position of the source
b) Vector of environmental parameters E

Factorization of the dependencies on source coordinates and environmental parameters:



$$\sum_{n=1}^{N^{(L)}} A_n u_n^{(L)}(z) e^{i\xi_n^{(L)} x}$$

$$\sum_{n=1}^{N^{(R)}} B_n u_n^{(R)}(z) e^{i\xi_n^{(R)} x}$$

Scattering matrix:

$$B_n = \sum_{m=1}^{N^{(L)}} S_{nm} A_m$$

$$\frac{\partial S_{nm}}{\partial \vec{r}_0} = 0 \quad , \quad \frac{\partial A_n}{\partial \vec{E}} = 0$$

Green function in terms of S-matrix:

$$G(x,z;x_0,z_0) = \frac{\exp(-i3\pi/4)}{\sqrt{8\pi\left(x-x_0\right)\left\langle\xi\right\rangle}} \sum_{n=1}^{N^{(L)}} \sqrt{\xi_n^{(L)}}\, u_n^{(L)}(z_0) \exp\left(-i\xi_n^{(L)} x_0\right)$$

$$\times \sum_{m=1}^{N^{(R)}} S_{nm} \sqrt{\xi_m^{(R)}}\, u_m^{(R)}(z) \exp\left(i\xi_m^{(R)} x\right)$$

Second moment of the field:

$$Corr\left(x_0,z_0;x_A,z_A;x_B,z_B\right) = p\left(x_0,z_0;x_A,z_A;\right)p^*\left(x_0,z_0;x_B,z_B\right) =$$

$$= \frac{1}{8\pi\left(x_A - x_0\right)^{1/2}\left(x_B - x_0\right)^{1/2}} \sum_{n,n'=1}^{N^{(L)}} \sqrt{\xi_n^{(L)}\xi_{n'}^{(L)}}\, u_n^{(L)}(z_0) u_{n'}^{(L)}(z_0) e^{-i\left(\xi_n^{(L)} - \xi_{n'}^{(L)}\right)x_0}$$

$$\times \sum_{m,m'=1}^{N^{(R)}} S_{nm}\left(S_{n'm'}\right)^* \sqrt{\xi_m^{(R)}\xi_{m'}^{(R)}}\, u_m^{(R)}(z) u_{m'}^{(R)}(z) e^{i\left(\xi_m^{(R)} x_A - \xi_{m'}^{(R)} x_B\right)}$$

All dependence on environmental parameters is contained in matrix:

$$\Sigma_{n,m;n',m'} = S_{n,m}\left(S_{n',m'}\right)^*$$

This is $N^2 \times N^2$ with N^4 elements altogether

For N=30 we are dealing with 1,000,000 entries

For moving source averaging over x_0

$$\langle Corr \rangle_{x_0} = \frac{1}{L}\int_0^L Corr\left(x_0, z_0; x_A, z_A; x_B, z_B\right) dx_0 \rightarrow$$

$$\rightarrow S_{n,m}\left(S_{n,m'}\right)^*$$

Now we have N^3 entries

Statistical properties of the averaged matrix

$$\left\langle \Sigma_{k,k'}(\vec{E}) \right\rangle = \left\langle S_{n,m} \left(S_{n',m'} \right)^* \right\rangle$$

$$k = (n,m)\,,\; k' = (n',m')$$

Can be characterized by its spectrum:

$$\lambda = \lambda_i$$

and appropriate eigenfunctions

$$\varphi_i = \varphi_{n,m}$$

Example: sensitivity of the average intensity on the environmental parameters

$$\int \left|\Psi(\vec{E}_1) - \Psi(\vec{E}_2)\right|^2 dz dz_0 = \frac{I_0}{8\pi\left(x - x_0\right)\bar{\xi}} \sum_{n,m} \left|S_{n,m}(\vec{E}_1) - S_{n,m}(\vec{E}_2)\right|^2$$

$$\int \left|\Psi(\vec{E})\right|^2 dz dz_0 = \frac{I_0}{8\pi\left(x - x_0\right)\bar{\xi}} N$$

Quantitative gauge of sensitivity:

$$\eta = \left(\int \left|\Psi(\vec{E}_1) - \Psi(\vec{E}_2)\right|^2 dz dz_0\right)^{1/2} \left(\int \left|\Psi(\vec{E})\right|^2 dz dz_0\right)^{-1/2} =$$

$$= \left(\frac{1}{N} \sum_{n,m} \left|S_{n,m}(\vec{E}_1) - S_{n,m}(\vec{E}_2)\right|^2\right)^{1/2}$$

Motivation for a statistical description of the uncertainty associated with horizontal refraction and medium time-dependence:

— range-dependence and cross-range variation of environmental parameters affect the acoustic field in a different manner

— being relatively weak, effects of the cross-range variations are not necessarily negligible

— these effects accumulate with range rather rapidly (typically, as third power of range) and lead to ***biases*** in signal travel time and modal phases

— signal frequency wander due to medium non-stationarity introduces uncertainty in estimates of target velocity

— lack of the detailed knowledge of cross-range environmental inhomogeneities is likely to remain, for the foreseeable future, an obstacle for the accurate prediction of the underwater acoustic field

— environmental measurements cannot be repeated fast enough along the propagation path to make a deterministic prediction of the frequency wander possible

Main oceanographic processes and geophysical features to be accounted for in developing a statistical description of 3-D and 4-D effects in underwater sound propagation:

— variable ocean surface

— bubble plumes created by breaking surface waves

— sound speed variations, including those associated with internal waves and internal tides

— ocean currents, including along-shore currents due to infragravity waves

— bottom topography

— inhomogeneities in geoacoustic properties of sediments

RAY TRAVEL TIME BIAS

$$n^2 = n_0^2 + \mathbf{,\ :\ ,}\ \mathbf{2} = \mathbf{2}_0 + \mathbf{,}\ \mathbf{2}_1 + \mathbf{,}^2\ \mathbf{2}_2 + \ldots$$

$$\theta_1(\boldsymbol{r}) = \frac{k_0}{2}\int_{x_1}^{x} \frac{\mu\, dx'}{n_0 \sin\alpha \cos\psi}\,, \quad \theta_2(\boldsymbol{r}) = -\frac{1}{2k_0}\int_{x_1}^{x} \frac{\left(\nabla\theta_1\right)^2 dx'}{n_0 \sin\alpha \cos\psi}$$

$$\delta T_{HR} = -\frac{1}{2}\int_{x_<}^{x_>} \frac{c\, dx}{(x - x_<)^2 |\sin\alpha|} \left(\int_{x_<}^{x} \frac{\partial c}{\partial y} \frac{(x' - x_<)\, dx'}{c^2 \sin\alpha} \right)^2$$

$$n_0 = n(x, 0, z) \quad \mathsf{Y} \quad \mathsf{:}\,(x, 0, z) \quad \mathsf{/}\ 0$$

Point source field in the adiabatic approximation

$$p(\boldsymbol{r}_2, z_2; \boldsymbol{r}_1, z_1) = \sum_n \left(\frac{1}{8\pi D} \right)^{1/2} f_n(z_1; \boldsymbol{r}_1) f_n(z_2; \boldsymbol{r}_2) \exp\left(i \int_{\boldsymbol{r}_1}^{\boldsymbol{r}_2} q_n \, ds - \frac{3\pi i}{4} \right)$$

2-D problem:

$$D = \text{const} \cdot q_n(x_1) q_n(x_2)$$

3-D problem, general result:

$$D = \frac{\partial(x, y)}{\partial(\tau, \psi_1)} = q_n(\boldsymbol{r}_2) \left[\left(\frac{\partial y}{\partial \psi_1} \right)_\tau \cos\psi_2 - \left(\frac{\partial x}{\partial \psi_1} \right)_\tau \sin\psi_2 \right]$$

“Almost straight” horizontal rays

$$q_n^2(\mathbf{r}) = k_0^2 + \epsilon\, g(\mathbf{r}), \quad |g|/k_0^2 \lesssim 1, \quad \epsilon \ll 1, \quad L = k_0^2/|\nabla g|$$

$$D = q_n(x_1, 0)\, q_n(x_2, 0) \left[\int_{x_<}^{x_>} \frac{dx}{q_n(x, 0)} - \int_{x_<}^{x_>} dx\,(x - x_<)(x_> - x)\, \frac{\partial^2}{\partial y^2} \frac{1}{q_n(x, 0)} \right] + O(\epsilon^2)$$

$$\theta(\mathbf{r}_2, \mathbf{r}_1) = \int_{x_<}^{x_>} q_n(x, 0)\, dx - \frac{1}{2\sqrt{q_n(x_1, 0)\, q_n(x_2, 0)}} \int_0^{|x_2 - x_1|} \frac{ds}{s^2} \left(\int_0^s \frac{\partial q_n}{\partial y}(x_< + a, 0)\, a\, da \right)^2 + O(\epsilon^3)$$

Weakly nonstationary waveguide:

$$\xi_n(\omega, \boldsymbol{r}, t) = \xi_n^{(0)}(\omega, \boldsymbol{r}) + \mu_n(\omega, \boldsymbol{r}, t), \quad \mu_n = O(\epsilon)$$

Eikonal equation: $(\nabla \psi)^2 = \xi_n^2(-\partial\psi/\partial t, \boldsymbol{r}, t), \quad \omega = -\partial\psi/\partial t$

Solutions to the eikonal equation

Exact:

$$\psi(\boldsymbol{r}, t) = \psi\left(\boldsymbol{r}_s,\, t - \int_{\boldsymbol{r}_s}^{\boldsymbol{r}} ds\, \frac{\partial \xi_n}{\partial \omega}\right) + \int_{\boldsymbol{r}_s}^{\boldsymbol{r}} ds \left(\xi_n - \omega \frac{\partial \xi_n}{\partial \omega}\right)$$

Frozen-medium approximation:

$$\psi_n^{Fr}(\boldsymbol{r}, t) = \int_{\boldsymbol{r}_s}^{\boldsymbol{r}} ds_0\, \xi_n\big(\omega_r, \boldsymbol{r}(s_0), t_i\big) - \omega_r t$$

Quasi-stationary approximation:

$$\psi_n^{Qs}(\boldsymbol{r}, t) = \int_{\boldsymbol{r}_s}^{\boldsymbol{r}} ds_0\, \xi_n\Big(\omega_r, \boldsymbol{r}(s_0), t(\boldsymbol{r}(s_0))\Big) - \omega_r t\,,$$

$$t\big(\boldsymbol{r}(s_0)\big) = t - \int_{\boldsymbol{r}(s_0)}^{\boldsymbol{r}_r} ds_1\, \frac{\partial \xi_n^{(0)}}{\partial \omega}\big(\omega_r, \boldsymbol{r}(s_1)\big)$$

Signal frequency variation

$$\omega(\boldsymbol{r}_R, t) = \omega_0 + \omega_0 \int_0^S ds_0 \frac{1}{c^2} \frac{\partial c}{\partial t}\bigg|_{\boldsymbol{r} = \boldsymbol{r}(s_0),\ t = t_0 + \int_0^{s_0} \frac{ds_1}{c_0}}$$

"Giant Doppler" observations

$$<(\delta f/f_0)^2> \sim r\, L\, \Omega^2 \left(\frac{\delta c}{c^2}\right)^2 \left(\frac{h}{H}\right)^2$$

r = 300 km, *L* = 1 km, *J* = 15 min, **c* = 0.5 m/s, *h* =100 m,

H = 2000m

$$\delta f/f_0 = 2 \cdot 10^{-3} \quad \leftrightarrow \quad V_r = 3\ m/s$$

Cross-disciplinary interaction sought:

Our research would benefit from interaction with group(s) involved in quantifying variations in bottom topography, sediment parameters as well as statistical characteristics of internal waves and bubble plumes in the littoral zone.

Study of the uncertainty due to 3-D and 4-D effects: Summary of the theoretical background

1. A technique has been developed to reduce 3-D and 4-D propagation problems to a set of 2-D problems that retains computational efficiency of the uncoupled azimuth and frozen medium approximations but extends their domains of validity and improves accuracy of propagation modeling.

2. Closed-form applicability conditions are obtained for the N×2-D approximation used in conjunction with the ray or adiabatic mode theory. Corrections to the N×2-D approximation predictions are found to be significant under conditions of some recent experiments in deep and shallow water.

3. Horizontal refraction <u>decreases</u> ray travel time and adiabtic mode phase. the travel time and phase biases are proportional to cross-range environmental gradients squared and increase as third power of range for large-scale inhomogeneities and as second power of range for small-scale

(random) inhomogeneities.

4. Unlike the frozen medium approximation, the quasi-stationary approximation is a sufficiently accurate and efficient approach to modeling low-frequency sound propagation in the time-dependent ocean.

5. Ocean non-stationarity, particularly internal waves, are likely to be responsible for the anomalous Doppler shifts observed in some tomographic experiments.